(Exported LaTeX printout without Maple V.3 worksheet input lines)

Analysis of trapped dust particle with HERA-e values
D. Kelly, DESY, November 14, 1995

Contents

1 General substitutions

2 Dust properties

For compariosn with (Zimmerman, 1994) an SiO_2 dust particle of radius $1\mu m$ is assumed, although there are indications (?) that something like 10 particles of equivalent total mass are involved:

$$A_{atom\,SiO2} = 60, Z_{atom\,SiO2} = 30, \rho_{SiO2} = 2400.000000\,\frac{kg\_}{m\_^3}, Rdust = .1000000000\,10^{-5}\,m\_$$

$$A = \frac{4}{3}\,\frac{\pi\,R^3\,\rho}{m_p}$$

For $1\mu m$ SiO_2 particle:

$$A_{SiO2\,r=1\ \mu m} = .6010377828\,10^{13}$$

$$m_{dust\,r=1\ \mu m} = .1005309649\,10^{-13}\,kg\_$$

$$R_{dust\,10\ particles} = .4641588833\,10^{-6}\,m\_$$

Table: SiO_2 **particle**

Radius [micro m]	mass [kg]	A
5.000e+00	1.257e-12	7.513e+14
2.500e+00	1.571e-13	9.391e+13
1.667e+00	4.654e-14	2.783e+13
1.250e+00	1.963e-14	1.174e+13
1.000e+00	1.005e-14	6.010e+12
8.333e-01	5.818e-15	3.478e+12
7.143e-01	3.664e-15	2.190e+12
6.250e-01	2.454e-15	1.467e+12
5.556e-01	1.724e-15	1.031e+12
5.000e-01	1.257e-15	7.513e+11
4.545e-01	9.441e-16	5.645e+11
4.167e-01	7.272e-16	4.348e+11
3.846e-01	5.720e-16	3.420e+11
3.571e-01	4.580e-16	2.738e+11
3.333e-01	3.723e-16	2.226e+11

3 Lifetime

Beam lifetime:

$$\frac{1}{\tau} = \frac{\sigma_{tot}\,c\,p}{k_B\,T}$$

Typical measured e^+ lifetime in HERA at $20mA$ gives a standard gas lifetime:

$$\tau_{gas}(\,20\,mA\,) = 8.5\,hr\_$$

Typical measured e^- lifetime in HERA including lifetime disruption:

$$\tau_{gas+disruption}(\,20\,mA\,) = 3.5\,hr\_$$

Contribution from disrupting object:

$$.1176470588\,\frac{1}{hr\_} + \frac{1}{\tau_{disruption}} = .2857142857\,\frac{1}{hr\_}$$

$$\tau_{disruption} = 5.950000000\,hr$$

3.1 Bremsstrahlung from residual gas particles

$$\frac{1}{\tau_{gas}} = \frac{16}{411}\,{r_e}^2\,Z\,(\,Z+1\,)\,c\,n_A \ln\left(\frac{1}{\Delta}\right) \ln\left(183\,\frac{1}{Z^{1/3}}\right)$$

Summing over all components with $\alpha_{i,j}$ the number of atoms j in ith type of molecule:

$$n_A = \sum_{i,j} \alpha_{i,j}\,n_i$$

with

$$n_i = \frac{p_i}{k_B\,T}$$

HERA: comparison with experiment:

On 22/8/94 average pressures as given by the ion getter pump current were measured to be $p = 10^{-9}\ mbar$ without current and $p = 3.2 \times 10^{-9}\ mbar$ with positron current 18 mA and lifetime $\tau = 8\ hr$. Assume an N_2-*equivalent* gas mixture:

$$n_{A,N_2} := 2\,\frac{p_{N_2}}{k_B\,T}$$

$$\tau_{N2,HERA}(\,0\,) = 51.54411621\,hr$$

$$\tau_{N2,HERA}(\,18\,mA\,) = 16.10753631\,hr$$

The pressure values estimated from the pump ion current may be out by a factor of about two

Table: Residual gas lifetime for N_2 in HERA

pressure [mbar]	lifetime [hr]
1.00e-09	51.54
2.00e-09	25.77
3.00e-09	17.18
4.00e-09	12.89
5.00e-09	10.31
6.00e-09	8.59
7.00e-09	7.36
8.00e-09	6.44
9.00e-09	5.73
1.00e-08	5.15

Above formula agrees with WB notes: in hPa:

$$\tau_{N_2,HERA}(\,0\,) = .0001855588184\,\frac{s}{p_{hPa}}$$

For H_2:

$$n_{A,H_2} := 2\,\frac{p_{H_2}}{k_B\,T}$$

$$\tau_{H_2,HERA} = 1293.723928\,hr$$

3.2 Bremsstrahlung in the field of the nuclei of a macroparticle

$$\frac{1}{\tau_{brem}} = \frac{8}{411} \frac{{r_e}^2 \ln\left(\frac{1}{\Delta}\right) \ln\left(183 \frac{1}{{Z_{atom}}^{1/3}}\right) c\, A_{dust}\, Z_{atom}\, (Z_{atom}+1)}{A_{atom}\, \pi\, \sigma_x\, \sigma_y\, C}$$

For an SiO_2 particle of radius $1\mu m$:

$$\tau_{brems\,SiO2,\, r\,=\,1\ mu} = 15.80680994$$

For an SiO_2 particle of mass $A = 4 \times 10^14$:

```
proc(b,unit) local str; str := unit._; evalf(b/str)*unit end
```

```
test := proc(b,unit) local str; str := unit._; evalf(b/str)*unit end
```

$$m\_^2$$

$$5000.\ mm$$

$$\tau_{brem\,SiO2,\ A=4E14} = .2375122499\, hr$$

Mass required to account for observed lifetime drop through Bremsstrahlung from dust nuclei alone:

$$A = .1596721008\, 10^{14}$$

Table: Lifetime due to Bremsstrahlung from SiO_2 nuclei

Rdust [micro m]	Adust	lifetime [hr]
5.000	7.513e+14	0.13
2.500	9.391e+13	1.01
1.667	2.783e+13	3.41
1.250	1.174e+13	8.09
1.000	6.010e+12	15.81
0.833	3.478e+12	27.31
0.714	2.190e+12	43.37
0.625	1.467e+12	64.74
0.556	1.031e+12	92.19
0.500	7.513e+11	126.45

3.3 Duststrahlung

Bremsstrahlung in the electric field of the charged particle:

$$\Psi(\,b\,) = \frac{1}{2} \frac{h\, c\, \gamma^3}{\pi\, \rho\, E_e}$$

$$\rho = \frac{E_e\, b}{c\, \gamma\, dp_{trans}}$$

$$dp_{trans} = \frac{1}{2} \frac{Q\, e^2}{\pi\, \varepsilon_0\, c\, b}$$

$$\Psi(\,b\,) = \frac{1}{4} \frac{h\, c\, \gamma^4\, Q\, e^2}{\pi^2\, {E_e}^2\, b^2\, \varepsilon_0}$$

In HERA at 27GeV:

$$\Psi(\,b\,) = .6075971441\, 10^{-17} \frac{{m\_}^2\, Q}{b^2}$$

Rate of photons per unit time:

$$\frac{5}{6}\frac{\sqrt{3}\,c\,\alpha\,\Psi(\,b\,)}{\lambda_e\,\gamma\,\sqrt{1+\Psi(\,b\,)^{2/3}}}$$

Approximate interaction time:

$$\frac{b}{c\,\gamma}$$

Average number of emitted photons per electron per revolution:

$$N_\gamma(\,b\,) = \frac{5}{6}\frac{b\,\sqrt{3}\,\alpha\,\Psi(\,b\,)}{\gamma^2\,\lambda_e\,\sqrt{1+\Psi(\,b\,)^{2/3}}}$$

$$\lambda_e = \frac{h}{\gamma\,m_e\,c}$$

In HERA:

$$N_\gamma(\,b\,) = 1.230925355$$

---

4 Electric field

4.1 Electric field of cylindrical Gaussian beam

Cylindrical Gaussian beam profile:

$$\mathit{profile} := r \to \frac{1}{2}\frac{\mathrm{e}^{\left(-1/2\,\frac{r^2}{\sigma^2}\right)}}{\sigma^2\,\pi}$$

Check normalisation:

$$\int_0^\infty 2\,\mathrm{profile}(\,r\,)\,\pi\,r\,dr = \lim_{r\to\infty-} -\mathrm{e}^{\left(-1/2\,\frac{r^2}{\sigma^2}\right)} + 1$$

$$2\;\mathrm{E}(\,r\,)\,\pi\,r\,l = \frac{q\int_0^r 2\,\pi\,x\,\mathrm{profile}(\,x\,)\,dx}{\varepsilon_0}$$

$$E_{Gauss} := (\,r, \sigma, l, q\,) \to -\frac{1}{2}\frac{q\left(\mathrm{e}^{\left(-1/2\,\frac{r^2}{\sigma^2}\right)} - 1\right)}{\pi\,r\,l\,\varepsilon_0}$$

4.2 Electric field for cylinderical beam profile

Inside:

$$\mathrm{q}(\,r\,) = \frac{r^2\,q}{R^2}$$

$$2\,\mathrm{Ein}(\,r\,)\,\pi\,r\,l = \frac{q\,r^2}{R^2\,\varepsilon_0}$$

Take $R^2 = 2\sigma^2$:

$$E_{cylindrical,inside} := (\,r, \sigma, l\,) \to \frac{1}{4}\frac{r\,q}{\pi\,l\,\sigma^2\,\varepsilon_0}$$

Outside:

$$2\,\mathrm{Eout}(\,r\,)\,r\,\pi\,l = \frac{q}{\varepsilon_0}$$

$$E_{cylinder,out} := (\,r, l, q\,) \to \frac{1}{2}\,\frac{q}{\pi\,r\,l\,\varepsilon_0}$$

Electric field due to cylindrical Gaussian beam at wall X:

$$29.17688155\,\frac{V}{m\_}$$

Electric field due to cylindrical Gaussian beam at wall Y:

$$59.95849160\,\frac{V}{m\_}$$

Electric field due to Clindrical beam at wall X:

$$29.17688155\,\frac{V}{m\_}$$

4.3 Two-dimensional Gaussian beam

After (Kheifets, 1976; Bassetti & Erskine, 1980).

Gaussian beam profile (Kheifets, 1976; Bassetti & Erskine, 1980):

$$\mathit{profile2D} := (\,x, y, \mathit{sigx}, \mathit{sigy}\,) \to \frac{1}{2}\,\frac{\mathrm{e}^{\left(-1/2\,\frac{x^2}{\mathrm{sigx}^2}\right)}\,\mathrm{e}^{\left(-1/2\,\frac{y^2}{\mathrm{sigy}^2}\right)}}{\mathit{sigx}\;\mathit{sigy}\;\pi}$$

Check normalisation:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{profile2D}(\,x, y, 1, 1\,)\,dx\,dy$$

$$1$$

We must solve:

$$\left(\frac{\partial^2}{\partial x^2}\;\mathrm{phi}(\,x, y\,)\right) + \left(\frac{\partial^2}{\partial y^2}\;\mathrm{phi}(\,x, y\,)\right) = \frac{1}{2}\,\frac{q\,\mathrm{e}^{\left(-1/2\,\frac{x^2}{\mathrm{sigx}^2}\right)}\,\mathrm{e}^{\left(-1/2\,\frac{y^2}{\mathrm{sigy}^2}\right)}}{\mathit{sigx}\;\mathit{sigy}\;\pi\,\varepsilon_0}$$

The solution is (Kheifets, 1976):

$$\phi(\,x, y\,) = \frac{1}{4}\,\frac{q\displaystyle\int_0^{\infty}\frac{\mathrm{e}^{\left(-\frac{x^2}{2\,\mathrm{sigx}^2+s}-\frac{y^2}{2\,\mathrm{sigy}^2+s}\right)}}{\mathrm{sqrt}(\,2\,\mathit{sigx}^2+s\,)\,\mathrm{sqrt}(\,2\,\mathit{sigy}^2+s\,)}\,ds}{\pi\,\varepsilon_0}$$

Maple can't confirm this directly.

$$\frac{1}{4}\,\frac{q\displaystyle\int_0^{\infty} -2\,\frac{\%1}{(\,2\,\mathit{sigx}^2+s\,)^{3/2}\,\sqrt{2\,\mathit{sigy}^2+s}} + 4\,\frac{x^2\,\%1}{(\,2\,\mathit{sigx}^2+s\,)^{5/2}\,\sqrt{2\,\mathit{sigy}^2+s}}\,ds}{\pi\,\varepsilon_0}$$

$$+\,\frac{1}{4}\,\frac{q\displaystyle\int_0^{\infty} -2\,\frac{\%1}{(\,2\,\mathit{sigy}^2+s\,)^{3/2}\,\sqrt{2\,\mathit{sigx}^2+s}} + 4\,\frac{y^2\,\%1}{(\,2\,\mathit{sigy}^2+s\,)^{5/2}\,\sqrt{2\,\mathit{sigx}^2+s}}\,ds}{\pi\,\varepsilon_0}$$

$$\%1 := \mathrm{e}^{\left(-\frac{x^2}{2\,\mathrm{sigx}^2+s}-\frac{y^2}{2\,\mathrm{sigy}^2+s}\right)}$$

Maple can't integrate the substituted form:

$$\int_r^1 \frac{\mathrm{e}^{\left(a^2\,(\,t^2-1\,)+b^2\,\left(1-\frac{1}{t^2}\right)\right)}}{t^2-1}\,dt = \int_r^1 \frac{\mathrm{e}^{\left(a^2\,(\,t^2-1\,)+b^2\,\left(1-\frac{1}{t^2}\right)\right)}}{t^2-1}\,dt$$

Gaussian beam electric field from ϕ (Bassetti & Erskine, 1980):

$$Egauss2D := (x, y) \rightarrow \frac{1}{4} e\, nEl \,\mathrm{sqrt}\left(2\frac{\pi}{{\sigma_x}^2 - {\sigma_y}^2}\right) \left(\mathrm{w}\left(\frac{x + I\,y}{\mathrm{sqrt}\,(2\,{\sigma_x}^2 - 2\,{\sigma_y}^2)}\right) - \mathrm{e}^{\left(-1/2\frac{x^2}{{\sigma_x}^2} - 1/2\frac{y^2}{{\sigma_y}^2}\right)} \mathrm{w}\left(\frac{\frac{x\,\sigma_y}{\sigma_x} + \frac{I\,y\,\sigma_x}{\sigma_y}}{\mathrm{sqrt}\,(2\,{\sigma_x}^2 - 2\,{\sigma_y}^2)}\right)\right) \Big/ (\pi\,\varepsilon_0\,C)$$

$$\frac{1}{4} \frac{e\,nEl\,\sqrt{2}\sqrt{\frac{\pi}{{\sigma_x}^2 - {\sigma_y}^2}} \left(\mathrm{w}\left(\frac{x + I\,y}{\sqrt{2\,{\sigma_x}^2 - 2\,{\sigma_y}^2}}\right) - \mathrm{e}^{\left(-1/2\frac{x^2}{{\sigma_x}^2} - 1/2\frac{y^2}{{\sigma_y}^2}\right)} \mathrm{w}\left(\frac{\frac{x\,\sigma_y}{\sigma_x} + \frac{I\,y\,\sigma_x}{\sigma_y}}{\sqrt{2\,{\sigma_x}^2 - 2\,{\sigma_y}^2}}\right)\right)}{\pi\,\varepsilon_0\,C}$$

Definition of the complex error function:

$$w1 := z \rightarrow \mathrm{e}^{(-z^2)}\,(1 - \mathrm{erf}(-I\,z))$$

Alternative definition might be more stable:

$$w2 := z \rightarrow \mathrm{e}^{(-z^2)}\,\mathrm{erfc}(-I\,z)$$

$$.3678794412 + .6071577060\,I,\ .4275835762$$

$$.3678794412 + .6071577060\,I,\ .4275835762$$

$$0,\ .1107046377$$

They give different answers when complex numbers are involved !

Electric field due to Gaussian beam at wall Y:

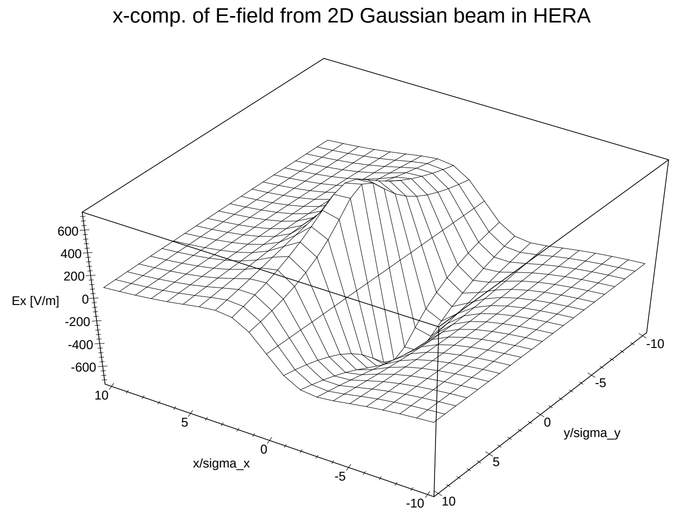
$$= 59.81751958\,\frac{V}{m\_}$$

Electric field due to Gaussian beam at wall X:

$$= \frac{(-.7825916713\,10^{-384} + 29.19326843\,I)\,V}{m\_}$$

which agree well with round-beam approximation.

FORTRAN routines are available to compute the complex error function:
- CWERF and WWERF in the CERN program library 'mathlib'.
- S15DDF in NAGLIB.



5 Magnetic field

Strong vertical fields are present in dipole regions:

$$Q\,e\,v\,B = \frac{m\,v^2}{\rho}$$

$$B = \frac{p}{\rho\,Q\,e}$$

HERA electron ring:

$$B_{e,HERA} = .09375080940\,T$$

Estimate the momentum from the horizontal oscillation frequency and the beam width for a $1\mu m$ trapped dust particle. Frequency and charge estimates from following sections and simulations.

$$p = m_{dust}\,\sigma_{x\,HERA}\,f_{x,dust}$$

$$\sigma_{x\,HERA} = .001000000000\,m\_,\ f_{x,dust} = 1497\,\frac{1}{s\_},\ m_{dust} = \frac{1}{10000000000000}\,kg\_$$

$$p = .1497000000\,10^{-12}\,\frac{kg\_\,m\_}{s\_}$$

Equate with vertical dipole B-field in electron ring to obtain the typical bending radius of an oscillating particle with charge number $Q = 5.7 \times 10^7$. The magnetic field may thus be ignored.

$$.093\,\frac{kg\_}{s\_^2\,A\_} = .1639216671\,\frac{kg\_\,m\_}{\rho\,s\_^2\,A\_}$$

$$\rho_{B,dust} = 1.762598571\,m\_$$

6 Longitudinal motion

6.1 Longitudinal motion in vertical B-field with Møller acceleration but without beam E-field

NB: What follows ignores the overwhelming E-field of the beam. One might also include the weak motion due to the Gaussian variation in beam cross-section during bunch passage.

Cycloid motion in (x, s) plane when Møller scattering (see subsection below) is included:

$$\frac{\partial^2}{\partial t^2}\,\mathrm{s}(t) = -\frac{q\,v_{perp}\,B\cos(\theta(t))}{m} + \delta,\ \frac{\partial^2}{\partial t^2}\,\mathrm{x}(t) = -\frac{q\,v_{perp}\,B\sin(\theta(t))}{m}$$

$$\frac{\partial^2}{\partial t^2}\,\mathrm{s}(t) = -\Omega\left(\frac{\partial}{\partial t}\,\mathrm{x}(t)\right) + \delta,\ \frac{\partial^2}{\partial t^2}\,\mathrm{x}(t) = \Omega\left(\frac{\partial}{\partial t}\,\mathrm{s}(t)\right)$$

With angular frequency (at B=0.1T):

$$\Omega = \frac{e\,Q\,B}{A\,m_p}$$

$$A = 100000\,Q,\ B = .1\ T\_$$

$$\Omega = 95.78830872\,\frac{1}{s\_}$$

$$\Omega_{Cycloid} := 95.78830872\,\frac{1}{s\_}$$

and Møller scattering acceleration:

$$\delta_{Moeller} := 12.0\,\frac{m\_}{s\_^2}$$

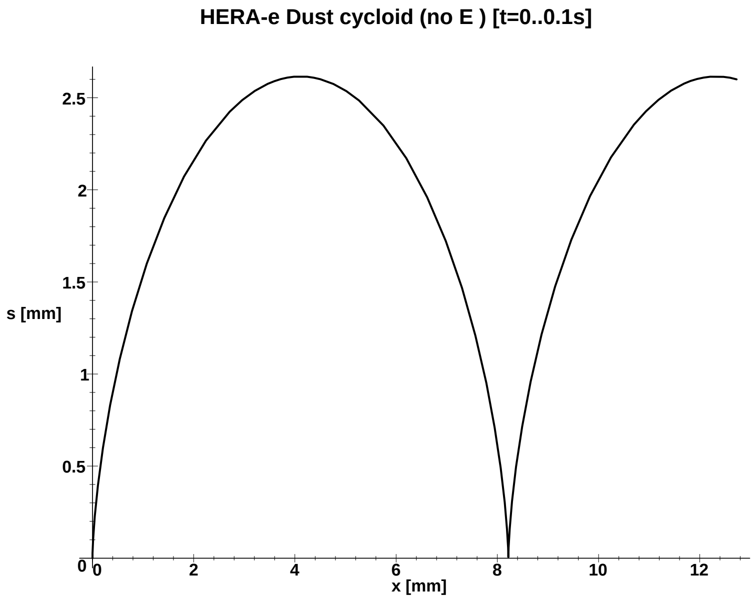
Cycloid solution for particle initially at rest and centre:

$$\mathrm{x}(0) = 0, \mathrm{s}(0) = 0, \mathrm{D}(x)(0) = 0, \mathrm{D}(s)(0) = 0$$

$$\left\{\mathrm{s}(t) = \frac{\delta}{\Omega^2} - \frac{\delta\cos(\Omega\,t)}{\Omega^2},\ \mathrm{x}(t) = \frac{\delta\,t}{\Omega} - \frac{\delta\sin(\Omega\,t)}{\Omega^2}\right\}$$

Numerical values:

$$\Omega = 95.78830872\,\frac{1}{s\_},\ \delta = 12.0\,\frac{m\_}{s\_^2}$$



Ignoring the overwhelming E-field the particle would leave the beam horizontally after about 0.01 s. The maximum longitudinal deflection would be:

$$2\,\frac{\delta}{\Omega^2} = .002615689754\,m\_$$

6.2 Longitudinal motion in vertical B-field, linear E-field (beam) and Møller scattering

Include the horizontal electric field (beam) in linear approximation together with vertical B-field and Møller scattering (δ). The longitudinal oscillation due to the Gaussian bunch profile is ignored:

$$\frac{\partial^2}{\partial t^2}\,\mathrm{s}(\,t\,) = -\Omega\,\left(\frac{\partial}{\partial t}\,\mathrm{x}(\,t\,)\right) + d,\ \frac{\partial^2}{\partial t^2}\,\mathrm{x}(\,t\,) = \Omega\,\left(\frac{\partial}{\partial t}\,\mathrm{s}(\,t\,)\right) - \nu^2\,\mathrm{x}(\,t\,)$$

where nu is the angular frequency due to the beam field.

An analytical solution (best method Laplace) exists. Initial conditions:

$$\mathrm{x}(\,0\,) = x0,\ \mathrm{D}(\,x\,)(\,0\,) = 0,\ \mathrm{s}(\,0\,) = 0,\ \mathrm{D}(\,s\,)(\,0\,) = 0$$

$$\Bigg\{\mathrm{s}(\,t\,) = \frac{\Omega^2\,d}{\%2} + \frac{t\,\Omega\,x0\,\nu^2}{\Omega^2+\nu^2} + \frac{1}{2}\,\frac{t^2\,d\,\nu^2}{\Omega^2+\nu^2} - \frac{\Omega\,x0\,\nu^4\sin(\,\%1\,)}{\%2\,\sqrt{\Omega^2+\nu^2}} - \frac{x0\,\Omega^3\,\nu^2\sin(\,\%1\,)}{\%2\,\sqrt{\Omega^2+\nu^2}} - \frac{\Omega^2\,d\cos(\,\%1\,)}{\%2},$$

$$\mathrm{x}(\,t\,) = \frac{x0\,\Omega^2}{\Omega^2+\nu^2} + \frac{t\,\Omega\,d}{\Omega^2+\nu^2} - \frac{\Omega\,d\sin(\,\%1\,)}{(\Omega^2+\nu^2)^{3/2}} + \frac{x0\,\nu^2\cos(\,\%1\,)}{\Omega^2+\nu^2}\Bigg\}$$

$$\%1 := \sqrt{\Omega^2+\nu^2}\,t$$

$$\%2 := \Omega^4 + 2\,\Omega^2\,\nu^2 + \nu^4$$

Clearly both longitudinal and horizontal coordinates always grow if ($x_0 = 0$), whence the solution becomes

$$\left\{ \mathrm{s}(t) = \frac{\Omega^2 d}{\Omega^4 + 2\,\Omega^2\,\nu^2 + \nu^4} + \frac{1}{2}\,\frac{t^2\,d\,\nu^2}{\Omega^2 + \nu^2} - \frac{\Omega^2\,d\cos\left(\sqrt{\Omega^2+\nu^2}\,t\right)}{\Omega^4 + 2\,\Omega^2\,\nu^2 + \nu^4},\ \mathrm{x}(t) = \frac{t\,\Omega\,d}{\Omega^2+\nu^2} - \frac{\Omega\,d\sin\left(\sqrt{\Omega^2+\nu^2}\,t\right)}{(\Omega^2+\nu^2)^{3/2}} \right\}$$

The situation is more complicated for general x_0, and stable oscillations may appear in the x-direction, or the oscillations may wander out of the beam quickly. The general asymptotes are:

$$x - asymptote = \frac{t\,\Omega\,d}{\Omega^2+\nu^2} + \mathrm{O}(1)$$

$$s - asymptote = \frac{1}{2}\,\frac{t^2\,d\,\nu^2}{\Omega^2+\nu^2} + \frac{t\,\Omega\,x0\,\nu^2}{\Omega^2+\nu^2} + \mathrm{O}(1)$$

In any case particle capture in pure dipoles in not assured.

Numerical values:

$$\Omega = 95.78830872\,\frac{1}{s\_},\ \nu = 18849.55592\,s\_,\ d = 12.0\,\frac{m\_}{s\_^2},\ x0 = .0003333333333\,m\_$$

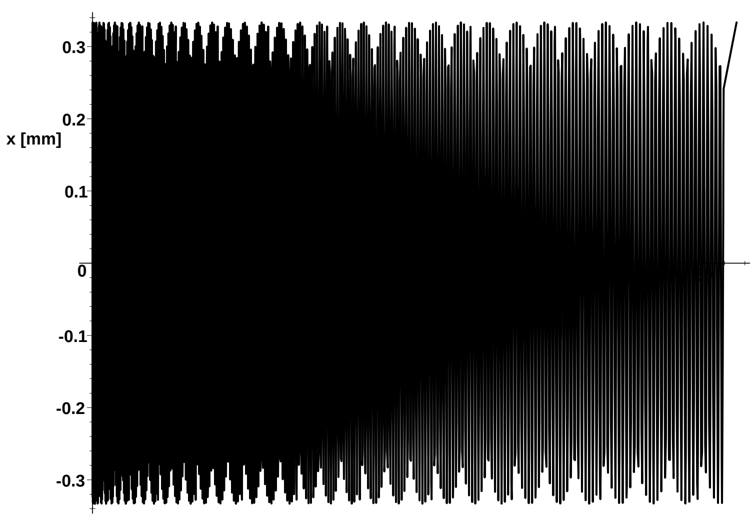
Time to reach $10\sigma_x$:

$$342.4744771\,s\_$$

Time to travel a FODO half-cell in HERA:

$$1.391790727\,s\_$$

HERA-e dust (x,s) plane (linear E) [t=0..0.1s]



6.3 Longitudinal motion in a quadrupole

6.3.1 Kick strength

A longitudinal kick will occur due to the variation in beam cross-section during a bunch's passage (as opposed to the intrinsic Gaussian variation), and this kick depends on the β-function gradient, i.e. on $\alpha = -d\beta/dt/2$. From (Zimmerman, 1994) the beam kick for one bunch crossing is:

$$\Delta_{bunch} = 2\,\frac{N_{el}\,c\,r_p\,Q\,\alpha_x\,\varepsilon_x}{n_{bunch}\,(\sigma_x+\sigma_y)\,A\,\sigma_x}$$

$$N_{el} = \frac{i\,T}{e}, T = \frac{C}{c}, r_p = \frac{1}{4}\,\frac{e^2}{\pi\,\varepsilon_0\,m_p\,c^2}$$

$$\Delta_{bunch} = \frac{1}{2}\,\frac{i\,C\,e\,Q\,\alpha_x\,\varepsilon_x}{\pi\,\varepsilon_0\,m_p\,c^2\,n_{bunch}\,(\sigma_x+\sigma_y)\,A\,\sigma_x}$$

Numerical values:

$$i = \frac{1}{50}\,A\_, \varepsilon_x = .1225221135\,10^{-6}\,m\_, n_{bunch} = 168$$

$$\Delta_{bunch} = 1.449643885\,\frac{m\_\,Q\,\alpha_x}{s\_\,A}$$

where $\alpha_x \approx 1$ in HERA, which is a fraction larger than FZ's estimate of $0.8Q/Am/s$ (per bunch) - probably due to a different emittance value based on FZ's average β-function of 27 m.

$$\varepsilon = \frac{{\sigma_x}^2}{\beta_{av}}$$

$$\varepsilon = .3703703704\,10^{-7}\,m\_$$

Averaged over the entire machine this gives, for $A/Q = 10^5$, the following average acceleration:

$$\left(\frac{\partial}{\partial t}\,\mathrm{s}(\,t\,)\right)_{\alpha_x} = \frac{\Delta_{bunch}\,n_{bunch}}{T}$$

$$\Delta_{bunch} = 1.449643885\,\frac{m\_\,Q\,\alpha_x}{s\_\,A}, A = 100000\,Q, n_{bunch} = 168, T = \frac{C}{c}$$

$$\left(\frac{\partial}{\partial t}\,\mathrm{s}(\,t\,)\right)_{\alpha_x} = 114.4201646\,\frac{m\_\,\alpha_x}{{s\_}^2}$$

6.3.2 Force and potential near a QP

The particle will tend to move towards the centre of horizontally defocussing quadrupoles. In a regular FODO cell α is linear in s and looks (downstream) something like

$$\alpha_{D,HERA} := s \rightarrow -6.78\,m\_\left(.085\,\frac{1}{m\_} + .0144\,\frac{s}{m\_}\right)$$

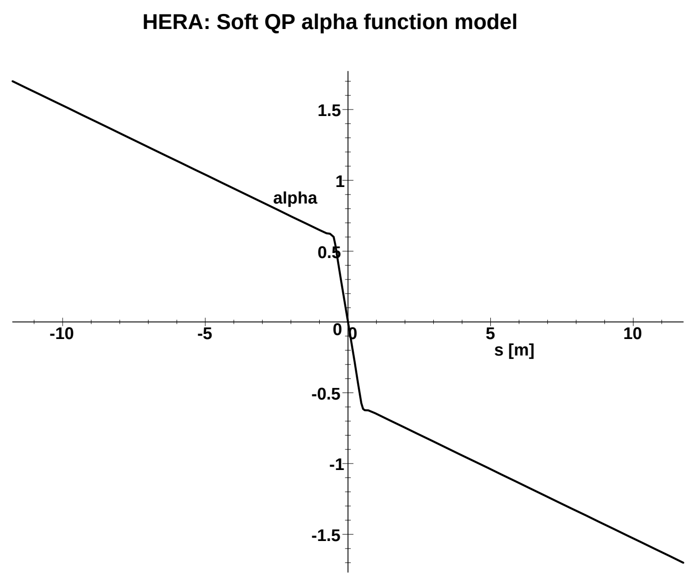
The simple FODO cell model assumes a discontinuity in α through an infinitely short QP. One can construct a thick lens model using a softened Heaviside function over length-scale λ:

$$Heavi := (\,s, \lambda\,) \rightarrow \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{s}{\lambda}\right)$$

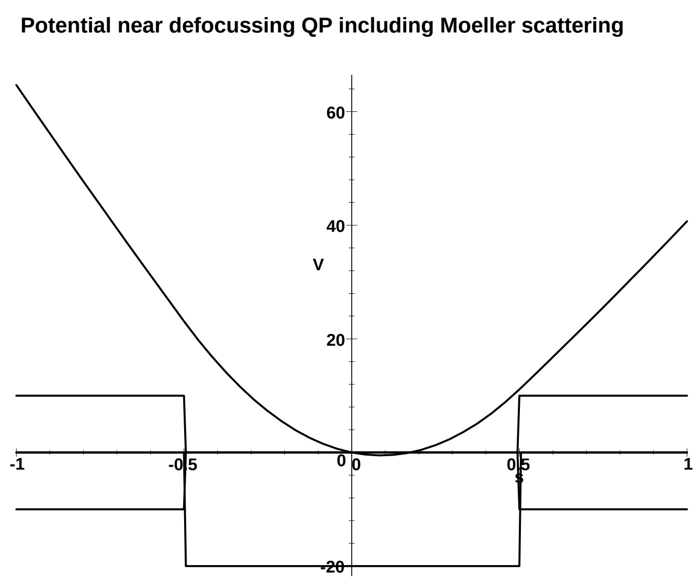
Define FODO cell properties:

$$dQP, amin, amax, lfodo$$

$$.5, .6, 1.7, 11.75550000$$



Compute the potential from a hard model. Maple can't integrate pieccewise procedure functions, so construct the potential from piecewise integrals (taking s=0 as reference):

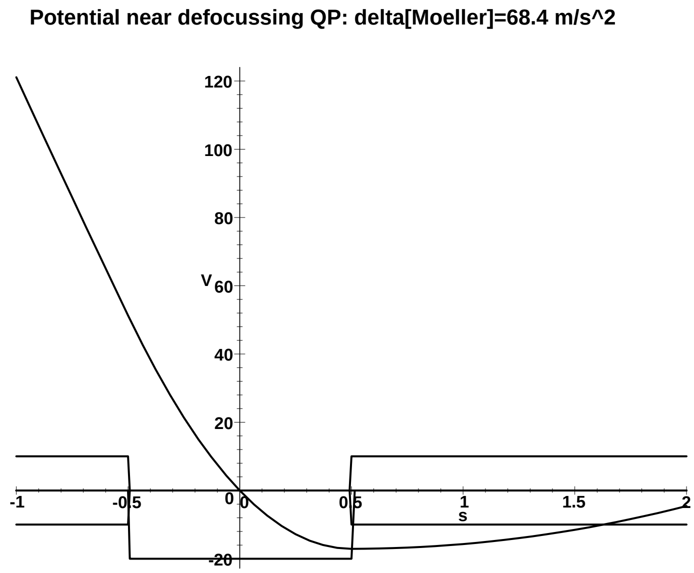


If the Møller scattering is larger than estimated, or if the ionisation of the dust particle is lower, then the potential may not be closed within the QP. Indeed a factor of about 6 error in either estimates is sufficient to assure escape. The condition for a potential bucket within the QP is (from the force equation):

$$114\,\alpha_{middle}(\,dQP,\,dQP,\,amin,\,amax,\,lfodo\,) + \delta = 0$$

$$\delta = 68.40000000$$

Of course there is always (to the extent of the FODO cell), a potential bucket. The question is then what happens to the particle outside the QP (i.e. in the dipole). The field can do no work on the particle, but introduces coupling between the s and x directions, and thus to the motion fue to the E-field of the beam.



6.3.3 D.E. solution near QP without dipole motion

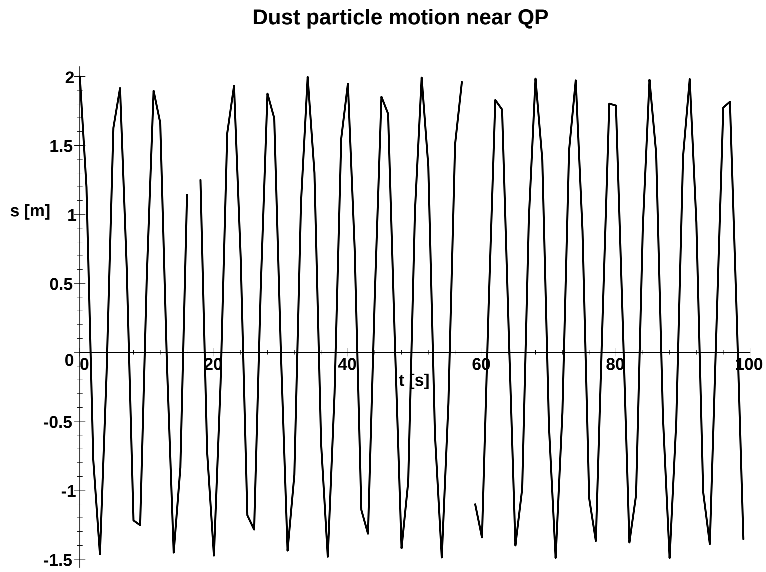
One can simulate the longitudinal d.e. based on the soft model, including Møller scattering, but ignoring the transverse E-field due to the beam and vertical B-field within the dipoles.

$$\frac{\partial^2}{\partial t^2}\,\mathrm{s}(\,t\,) = 114\,\alpha_{soft}\left(\mathrm{s}(\,t\,),\,dQP,\,amin,\,amax,\,lfodo,\,\frac{1}{10}\right) + 12$$

$$\mathrm{s}(\,0\,) = 2,\,\mathrm{D}(\,s\,)(\,0\,) = 0$$

```
proc(rkf45_x) ... end
```

$$\left[t = 1,\mathrm{s}(\,t\,) = 1.197202620613907,\frac{\partial}{\partial t}\,\mathrm{s}(\,t\,) = -10.49798225714339\right]$$



6.3.4 (x,s) D.E. solution near QP including E_x, B_z

Numerical solution is possible with the aid of an approximate Heaviside function (to define QP and dipole boundaries), but the evaluation is lengthy.

$$Byf := s \to 95\,\mathrm{Heavi}\left(s - dQP, \frac{1}{100}\right) + 95\,\mathrm{Heavi}\left(-s - dQP, \frac{1}{100}\right)$$

$$\mathrm{diff}(\,\mathrm{s}(\,t\,), t\,\$\,2\,) = -\left(\frac{\partial}{\partial t}\,\mathrm{x}(\,t\,)\right)\,\mathrm{Byf}(\,\mathrm{s}(\,t\,)\,) + 114\,\alpha_{soft}\left(\mathrm{s}(\,t\,), dQP, amin, amax, lfodo, \frac{1}{10}\right) + 12,$$

$$\mathrm{diff}(\,\mathrm{x}(\,t\,), t\,\$\,2\,) = \left(\frac{\partial}{\partial t}\,\mathrm{s}(\,t\,)\right)\,\mathrm{Byf}(\,\mathrm{s}(\,t\,)\,) - 36000000\,\pi^2\,\mathrm{x}(\,t\,)$$

$$\mathrm{s}(\,0\,) = 2, \mathrm{D}(\,s\,)(\,0\,) = 0, \mathrm{x}(\,0\,) = .0003333333333, \mathrm{D}(\,x\,)(\,0\,) = 0$$

```
proc(rkf45_x) ... end
```

6.4 Elastic scattering

Scattering of an electron from a fixed point charge (second order theory):

$$\frac{\partial}{\partial \Omega}\,\sigma = \frac{1}{64}\,\frac{Z^2\,e^4\left(1 - \beta^2 \sin\left(\frac{1}{2}\,\theta\right)^2 + \frac{1}{137}\,Z\,\pi\,\beta \sin\left(\frac{1}{2}\,\theta\right)\left(1 - \sin\left(\frac{1}{2}\,\theta\right)\right)\right)}{\pi^2\,{\varepsilon_0}^2\,p^2\,\beta^2 \sin\left(\frac{1}{2}\,\theta\right)^4}$$

Electron-positron (Bhabha) scattering $e^+e^- \to e^+e^-$: lowest order theory:

$$\frac{\partial}{\partial\Omega}\sigma = \frac{1}{8}{r_e}^2 c^2 {m_e}^2 \left(\frac{1}{4}\frac{1+\cos\left(\frac{1}{2}\theta\right)^4}{\sin\left(\frac{1}{2}\theta\right)^4} + \frac{1}{8} + \frac{1}{8}\cos(\theta)^2 - \frac{1}{16}\frac{\cos\left(\frac{1}{2}\theta\right)^4}{\sin\left(\frac{1}{2}\theta\right)^4} \right)$$

Electron-electron scattering (Møller) scattering: for $p \gg m_e$:

$$\frac{\partial}{\partial\Omega}\sigma = \frac{1}{2}\frac{{r_e}^2 c^2 {m_2}^2 \left(3+\cos(\theta)^2\right)^2}{p^2 \sin(\theta)^4}$$

7 Particle frequency estimates

NB: These are estimates of the average oscillation frequency for large dust particles not the momentary frequency during bunch passage (only relevant for kicking ions).

My `dustde.f` simulation using the cylindrical Gaussian beam gives $Q \approx 10^7$. A simulation using the correct two-dimensional Gaussian beam and including the charge profile in the charge d.e. gives $Q \approx 5.7 \times 10^{06}$. The (Zimmerman, 1994) approximation seems to oversestimate the avergae electron flux seen by the particle.

7.1 Frequency for Gaussian cylindrical beam

$$fgauss := (R, \sigma) \rightarrow \frac{1}{2}\frac{\mathrm{sqrt}\left(\frac{Q\,e\,\mathrm{Egauss}(R,\sigma)}{m_p\,A\,R}\right)}{\pi}$$

$$fx = .1206136670\,10^7 \frac{\sqrt{Q}}{s\_\sqrt{A}}$$

$$fx = 1555.770866 \frac{1}{s\_}$$

$$fy = .5244072480\,10^7 \frac{\sqrt{Q}}{s\_\sqrt{A}}$$

$$fy = 6764.221159 \frac{1}{s\_}$$

7.2 Frequency for two-dimensional beam

$$fgauss2DX := (x, y) \rightarrow \frac{1}{2}\frac{\mathrm{sqrt}\left(\frac{Q\,e\,\Im(\mathrm{Egauss2D}(x,y))}{m_p\,A\,x}\right)}{\pi}$$

$$fgauss2DY := (x, y) \rightarrow \frac{1}{2}\frac{\mathrm{sqrt}\left(\frac{Q\,e\,\Re(\mathrm{Egauss2D}(x,y))}{m_p\,A\,y}\right)}{\pi}$$

$$fx := (Q, A) \rightarrow .1537997339\,10^7 \frac{\sqrt{Q}}{s\_\sqrt{A}}$$

$$fy := (Q, A) \rightarrow .3206912693\,10^7 \frac{\sqrt{Q}}{s\_\sqrt{A}}$$

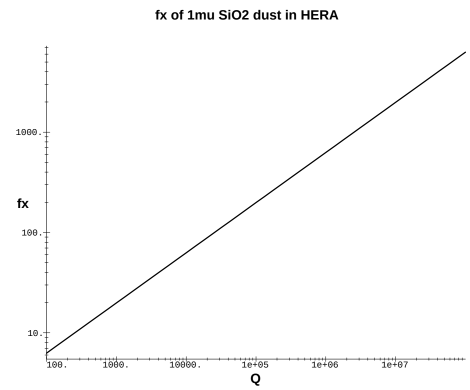
$$fx(10^{7}) = 1983.831113 \frac{1}{s\_}$$

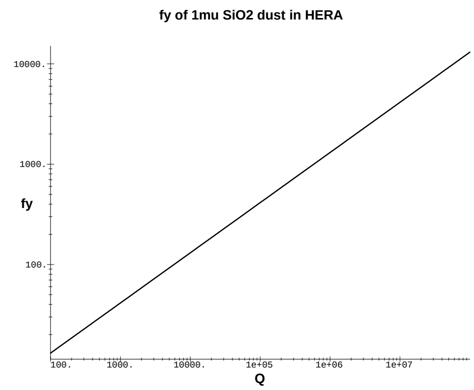
$$fy(10^{7}) = 4136.530680 \frac{1}{s\_}$$

`dustde.f` simulation values:

$$fx(5.7.10^{6}) = 1497.759645 \frac{1}{s\_}$$

$$fy(5.7.10^{6}) = 3123.012176 \frac{1}{s\_}$$





8 Particle charge development

After (Zimmerman, 1994):

8.1 Ionisation

$$\frac{\partial^2}{\partial x\, \partial T} \mathrm{N}(t, x) = 2 \frac{\pi N_A {r_e}^2 m_e c^2 Z_{atom}}{A_{atom} T^2}$$

$$\frac{\partial}{\partial x} N_{escape}(x) = \int_{1/4}^{\infty} \frac{\mathrm{Q}(t) e^2}{\pi \varepsilon_0 R} \mathrm{rhs}\left(\frac{\partial^2}{\partial x\, \partial T} \mathrm{N}(t, x) = 2 \frac{\pi N_A {r_e}^2 m_e c^2 Z_{atom}}{A_{atom} T^2} \right) dT$$

$$\frac{\partial}{\partial x} N_{escape}(x) = 8 \frac{\pi^2 N_A {r_e}^2 m_e c^2 Z_{atom} \varepsilon_0 R}{A_{atom} \mathrm{Q}(t) e^2}$$

Individual electron passage length:

$$\mathrm{d}(z) = 2 \sqrt{R^2 - z^2}$$

Average electron passage length:

$$\frac{\int_0^{2\pi} \int_0^R 2 z \sqrt{R^2 - z^2}\, dz\, d\phi}{\int_0^{2\pi} \int_0^R z\, dz\, d\phi} = \frac{4}{3} R$$

Average number of ionisations **per individual electron passage**:

$$Nescape := \frac{32}{3} \frac{\rho R^2 \pi^2 N_A {r_e}^2 m_e c^2 Z_{atom} \varepsilon_0}{A_{atom} \mathrm{Q}(t) e^2}$$

8.1.1 Approximation: average experience of particle in beam

As used in (Zimmerman, 1994):

Fraction of electrons passing through dust particle:

$$\frac{1}{2}\frac{R^2}{\sigma_x\,\sigma_y}$$

Electron flux (NB: the bunch structure is ignored for dust particles):

$$N_{el}\,f_{rev} = \frac{\mathrm{i}(\,t\,)}{e}$$

Beam-averaged ionisation rate:

$$\frac{\partial}{\partial t}\,Q_{ioniz}(\,t\,) = \frac{1}{2}\,\frac{Nescape\,\mathrm{i}(\,t\,)\,R^2}{e\,\sigma_x\,\sigma_y}$$

$$\frac{\partial}{\partial t}\,Q_{ioniz}(\,t\,) = \frac{16}{3}\,\frac{\rho\,R^4\,\pi^2\,N_A\,{r_e}^2\,m_e\,c^2\,Z_{atom}\,\varepsilon_0\,\mathrm{i}(\,t\,)}{A_{atom}\,\mathrm{Q}(\,t\,)\,e^3\,\sigma_x\,\sigma_y}$$

HERA examples for (Zimmerman, 1994) ionisation approximation

For an arbitray dust particle at any current:

$$\frac{\partial}{\partial t}\,Q_{ioniz}(\,t\,) = .1929002026\,10^{39}\,\frac{\rho\,R^4\,Z_{atom}\,\mathrm{i}(\,t\,)}{kg\_m\_s\_A\_A_{atom}\,\mathrm{Q}(\,t\,)}$$

For a $1\mu m$ SiO_2 particle at any current:

$$\frac{\partial}{\partial t}\,Q_{ioniz}(\,t\,) = .1929002026\,10^{39}\,\frac{rhodust\,Rdust^4\,Zatom\,\mathrm{i}(\,t\,)}{kg\_m\_s\_A\_Aatom\,\mathrm{Q}(\,t\,)}$$

For a $1\mu m$ SiO_2 particle at any current::

$$\frac{\partial}{\partial t}\,Q_{ioniz}(\,t\,) = .2314802431\,10^{18}\,\frac{\mathrm{i}(\,t\,)}{s\_A\_\mathrm{Q}(\,t\,)}$$

Comparison with (Zimmerman, 1994): SiO_2 particle radius R at $20mA$ in HERA:

$$\frac{\partial}{\partial t}\,Q_{ioniz}(\,t\,) = .4629604861\,10^{40}\,\frac{R^4}{m\_^4\,s\_\mathrm{Q}(\,t\,)}$$

$$dQion := Q \to .4629604861\,10^{16}\,\frac{1}{s\_Q}$$

8.1.2 Ionisation including 2D beam profile

Fraction of electrons passing through dust particle at position (x, y):

$$\pi\,R^2\,\mathrm{profile2D}\,(\mathrm{x}(\,t\,), \mathrm{y}(\,t\,), \sigma_x, \sigma_y)$$

$$\frac{1}{2}\,\frac{R^2\,\mathrm{e}^{\left(-1/2\,\frac{\mathrm{x}(\,t\,)^2}{{\sigma_x}^2}\right)}\,\mathrm{e}^{\left(-1/2\,\frac{\mathrm{y}(\,t\,)^2}{{\sigma_y}^2}\right)}}{\sigma_x\,\sigma_y}$$

Ionisation rate:

$$\frac{\partial}{\partial t}\,Q_{ioniz}(\,t\,) = \frac{1}{2}\,\frac{Nescape\,R^2\,\mathrm{e}^{\left(-1/2\,\frac{\mathrm{x}(\,t\,)^2}{{\sigma_x}^2}\right)}\,\mathrm{e}^{\left(-1/2\,\frac{\mathrm{y}(\,t\,)^2}{{\sigma_y}^2}\right)}\,\mathrm{i}(\,t\,)}{\sigma_x\,\sigma_y\,e}$$

$$\frac{\partial}{\partial t}\,Q_{ioniz}(\,t\,) = \frac{16}{3}\,\frac{\rho\,R^4\,\pi^2\,N_A\,{r_e}^2\,m_e\,c^2\,Z_{atom}\,\varepsilon_0\,\mathrm{e}^{\left(-1/2\,\frac{\mathrm{x}(\,t\,)^2}{{\sigma_x}^2}\right)}\,\mathrm{e}^{\left(-1/2\,\frac{\mathrm{y}(\,t\,)^2}{{\sigma_y}^2}\right)}\,\mathrm{i}(\,t\,)}{A_{atom}\,\mathrm{Q}(\,t\,)\,e^3\,\sigma_x\,\sigma_y}$$

HERA examples for 2D profile ionisation

For an arbitrary dust particle at any current:

$$\frac{\partial}{\partial t} Q_{ioniz}(t) = .1929002026\,10^{39} \frac{\rho\, R^4\, Z_{atom}\, e^{\left(-500000.0000 \frac{x(t)^2}{m\_^2}\right)}\, e^{\left(-.945179584010^7 \frac{y(t)^2}{m\_^2}\right)}\, i(t)}{kg\_\, m\_\, s\_\, A\_\, A_{atom}\, Q(t)}$$

For a $1\mu m$ SiO_2 particle at any current:

$$\frac{\partial}{\partial t} Q_{ioniz}(t) = .2314802431\,10^{18} \frac{e^{\left(-500000.0000 \frac{x(t)^2}{m\_^2}\right)}\, e^{\left(-.945179584010^7 \frac{y(t)^2}{m\_^2}\right)}\, i(t)}{s\_\, A\_Q(t)}$$

Comparison with (Zimmerman, 1994): SiO_2 particle radius R at 20mA in HERA:

$$\frac{\partial}{\partial t} Q_{ioniz}(t) = .4629604864\,10^{40} \frac{R^4\, e^{\left(-500000.0000 \frac{x(t)^2}{m\_^2}\right)}\, e^{\left(-.945179584010^7 \frac{y(t)^2}{m\_^2}\right)}\, A\_(t)}{m\_^4\, s\_\, A\_Q(t)}$$

8.2 Deionisation

8.2.1 Deionisation through photoelectron capture

$$\frac{\partial}{\partial t} Q_{disc}(t) = -\frac{\sigma_{pe}}{\tau_{pe}\, \pi\, d\, C}$$

$$\frac{1}{\tau_{pe}} = \frac{1}{100} \frac{\mu\, \gamma\, c\, N_{el}}{\rho_{bend,av}}$$

$$\sigma_{pe} = \frac{\pi\, R^2\, T_{min} \ln\left(\frac{E_{pe,max}}{E_{pe,min}}\right)}{E_{pe,max}}$$

$$\frac{\partial}{\partial t} Q_{disc}(t) = -\frac{1}{400} \frac{\mu\, \gamma\, c\, i(t)\, e\, R\, Q \ln\left(\frac{E_{pe,max}}{E_{pe,min}}\right)}{f_{rev}\, \rho_{bend,av}\, \pi\, \varepsilon_0\, E_{pe,max}\, d\, C}$$

$$\frac{\partial}{\partial t} Q_{disc}(t) = -.2600627652\,10^{15} \frac{\gamma\, c\, i(t)\, e\, R\, Q\, s\_^2 \ln(1000.000000)}{f_{rev}\, \rho_{bend,av}\, \pi\, \varepsilon_0\, kg\_\, m\_^3\, C}$$

$$\frac{\partial}{\partial t} Q_{disc}(t) = -.3435188920\,10^{13} \frac{i(t)\, R\, Q}{s\_\, A\_\, Circumference\_H}$$

$$\frac{\partial}{\partial t} Q_{disc}(t) = -.5383464849\,10^{9} \frac{i(t)\, R\, Q}{s\_\, A\_\, m\_}$$

$$\frac{\partial}{\partial t} Q_{disc}(t) = -.1076692970\,10^{8} \frac{R\, Q}{s\_\, m\_}$$

Zimmermann (1994) gives (ignoring details of the photoelectron history in the magnetic field of the chamber)

$$\frac{\partial}{\partial t} Q_{disc}(t) = -.170000000\,10^{8} \frac{Q(t)\, R}{s\_\, m\_}$$

Seem to disagree: - Different numerical values for the bending radius ? Doesn't seem to explain values.
- Typesetting error 1.07 -¿ 1.7 ?
Until further notice use (Zimmerman, 1994) value, since photoelctron capture doesn't dominate anyway.

8.2.2 Deionisation through field-charge evaporation

$$-\frac{\int_0^T \int_0^{T2} \frac{1}{T1^2}\, dT1\, C_p\, dT2}{k_B} = -\frac{\infty\, C_p\, T}{k_B}$$

Approximate with lower limit T_0:

$$\frac{C_p \ln\left(\frac{T}{T_0}\right)}{k_B}$$

$$\frac{\partial}{\partial t} Q_{ev}(t) = -8\, \frac{A_{atom}\, m_p\, \pi^2\, R^2\, {k_B}^2\, T^2\, \mathrm{e}^{\left(-\frac{U+V-\Phi}{k_B T} + 1/4 \frac{e^2 \sqrt{Q(t)}}{\pi\, \varepsilon_0\, R\, k_B\, T} - \frac{C_p \ln\left(\frac{T}{T_0}\right)}{k_B}\right)}}{h^3}$$

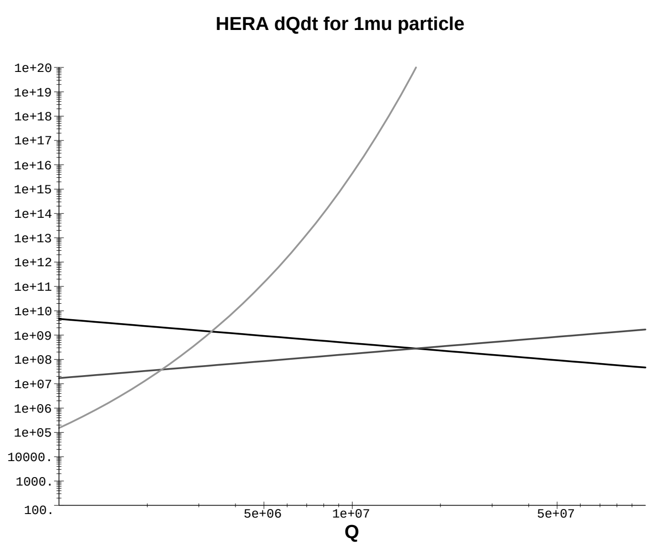
Example: HERA with **titanium** ((Zimmerman, 1994) doesn't have the work functions listed for SiO_2):

For a $1\mu m$ Ti particle at $1500K$:

$$dQfield := Q \to -.9345757880\, 10^{25}\, \mathrm{e}^{\left(-56.93916406 + .01114000181 \sqrt{Q}\right)}$$

e.g at $Q = 10^7$

$$-.3479202757\, 10^{16}$$



NB: SiO_2 values have been used for ionisation and disc., whereas Ti is used for the field evaporation.

8.3 Charge d.e. with HERA numerical values for arbitrary particle

Ionisation including variable current and beam profile:

$$dQdti := .1929002026\, 10^{39}\, \frac{\rho\, R^4\, Z_{atom}\, \mathrm{e}^{\left(-500000.0000 \frac{x(t)^2}{m\_^2}\right)}\, \mathrm{e}^{\left(-.9451795840\, 10^7 \frac{y(t)^2}{m\_^2}\right)}\, i(t)}{kg\_\, m\_\, s\_\, A\_\, A_{atom}\, Q(t)}$$

Deionisation:

$$dQdtd := -.170000000\,10^{8}\,\frac{\mathrm{Q}(t)\,R}{s\_\,m\_} - .9345757880\,10^{25}\,\mathrm{e}^{\left(-56.93916406+.01114000181\sqrt{\mathrm{Q}(t)}\right)}$$

Ionisation occurs only during bunch passage, but this is accounted for by the definition of average flux:

$$N_{el}\,f_{rev} = \frac{\mathrm{i}(t)}{e}$$

Photodeionisation should be reasonably independent of the position.

The d.e. then reads:

$$\frac{\partial}{\partial t}\,\mathrm{Q}(t) = \left(\frac{\partial}{\partial t}\,\mathrm{Q}(t)\right)_{ion} + \left(\frac{\partial}{\partial t}\,\mathrm{Q}(t)\right)_{deion}$$

$$\frac{\partial}{\partial t}\,\mathrm{Q}(t) = dQdti + dQdtd$$

$$deqnQ := \frac{\partial}{\partial t}\,\mathrm{Q}(t) = .1929002026\,10^{39}\,\frac{\rho\,R^4\,Z_{atom}\,\mathrm{e}^{\left(-500000.0000\,\frac{\mathrm{x}(t)^2}{\mathrm{m}\_^2}\right)}\,\mathrm{e}^{\left(-.9451795840\,10^{7}\,\frac{\mathrm{y}(t)^2}{\mathrm{m}\_^2}\right)}\,\mathrm{i}(t)}{kg\_\,m\_\,s\_\,A\_\,A_{atom}\,\mathrm{Q}(t)}$$
$$- .170000000\,10^{8}\,\frac{\mathrm{Q}(t)\,R}{s\_\,m\_} - .9345757880\,10^{25}\,\mathrm{e}^{\left(-56.93916406+.01114000181\sqrt{\mathrm{Q}(t)}\right)}$$

9 Sources of damping

Estimate of the total energy required to account for the electron loss through scattering:

$$N_{el}(t) = \frac{\mathrm{i}(t)}{f_{rev}\,e}$$

$$\frac{\partial}{\partial t}\,N_{el}(t) = -\frac{\mathrm{i}(t)\left(\frac{1}{\tau_{gas}} + \frac{1}{\tau_{dust}}\right)}{f_{rev}\,e}$$

Assume $\tau_{gas} \approx \tau_{dust} = 8h$ and average energy loss $= E\Delta E/E$:

$$\frac{\partial}{\partial t}\,\mathrm{E}(t) = -\frac{27}{100}\,\frac{GeV\_\mathrm{i}(t)\left(\frac{1}{\tau_{gas}} + \frac{1}{\tau_{dust}}\right)}{f_{rev}\,e}$$

$$\frac{\partial}{\partial t}\,\mathrm{E}(t) = -5746.875726\,\frac{kg\_\,m\_^2\,\mathrm{i}(t)\left(\frac{1}{\tau_{gas}} + \frac{1}{\tau_{dust}}\right)}{s\_^2\,A\_}$$

$$\frac{\partial}{\partial t}\,\mathrm{E}(t) = -.007981771842\,\frac{kg\_\,m\_^2}{s\_^3}$$

Some of this energy may damp the transverse motion of the particle.

9.1 Artificial excitation

The particle can be excited by kicking with the transverse feedback kicker. This frequency may be swept. All frequencies below are in true cycles per second (period $T = 1/f$).

Frequency of the transverse kicker:

$$fkick := t \to \frac{1}{2} f1 + \frac{1}{2} f2 + \frac{1}{2} (f2 - f1) \cos(2\pi\, fsweep\, t)$$

$$f2, f2, f1, \frac{1}{2} f1 + \frac{1}{2} f2$$

Actual displacement of the beam achieved:

$$Rkick := t \to kick \sin(2\pi\, \mathrm{fkick}(t)\, t)$$

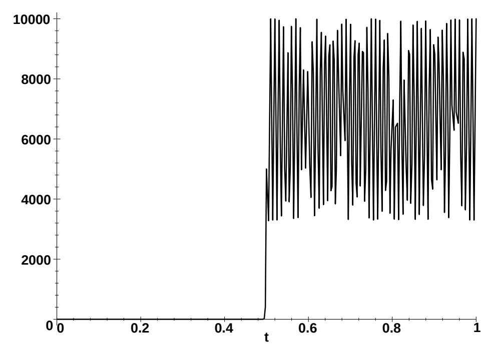
$$0$$

Include frequency evolution of kick in both directions in d.e. system (to enable adaptive solution):

$$fde := \frac{\partial}{\partial t} \mathrm{Rk}(t) = kick \cos\left(2\pi \left(\frac{1}{2} f1 + \frac{1}{2} f2 + \frac{1}{2} (f2 - f1) \cos(2\pi\, fsweep\, t)\right) t\right) \left(-2\pi^2 (f2 - f1) \sin(2\pi\, fsweep\, t)\, fsweep\, t + 2\pi \left(\frac{1}{2} f1 + \frac{1}{2} f2 + \frac{1}{2} (f2 - f1) \cos(2\pi\, fsweep\, t)\right)\right)$$

Set onset of kicking after some time (say 0.5 s) through approximate Heaviside function:

$$Heavi := (x, l) \to \frac{1}{2} + \frac{1}{2} \tanh\left(\frac{x}{l}\right)$$



Fortran output to dustde.f. Set frequency a little lower in x through factor *fxtoy*.

10 Equation of motion

10.1 EOM for cylindrical Gaussian beam without charge development

Forces without transverse kick excitation:

$$ForceX := R \to -e\_\mathrm{Q}(t)\, \mathrm{Egauss\_H}(R, SigmaX\_H)$$

$$ForceY := R \to -e\_\mathrm{Q}(t)\, \mathrm{Egauss\_H}(R, SigmaY\_H)$$

$$deqnX := \mathrm{diff}(\mathrm{R}(t), t\,\$\,2) = \frac{\mathrm{ForceX}(\mathrm{R}(t)\, m\_)\, s\_^2}{m\, m\_}$$

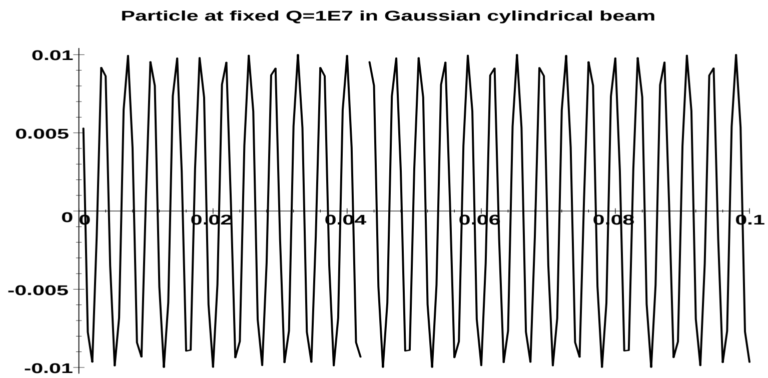
$$\frac{\partial^2}{\partial t^2} \mathrm{R}(t) = -.1921282720\, 10^{-18} \frac{\mathrm{Q}(t) \left(-\mathrm{e}^{(-500000.0000\, \mathrm{R}(t)^2)} + 1\right) kg\_}{\mathrm{R}(t)\, m}$$

$$deqnY := \mathrm{diff}(\mathrm{R}(t), t\,\$\,2) = \frac{\mathrm{ForceY}(\mathrm{R}(t)\, m\_)\, s\_^2}{m\, m\_}$$

$$\frac{\partial^2}{\partial t^2}\,\mathrm{R}(t) = -.1921282720\,10^{-18}\,\frac{\mathrm{Q}(t)\,\left(-\mathrm{e}^{\left(-.945179584010^{7}\,\mathrm{R}(t)^2\right)}+1\right)\,kg\_}{\mathrm{R}(t)\,m}$$

Numerical solution for Cylindrical Gaussian beam without charge development:

```
F := proc(rkf45_x) ... end
```



10.2 Transverse EOM for Cylindrical Gaussian beam with charge development

$$\mathrm{diff}(\,\mathrm{R}(t), t\,\$\,2\,) = \frac{\mathrm{ForceY}(\,\mathrm{R}(t)\,m\_)\,s\_^2}{m\,m\_}$$

$$\frac{\partial^2}{\partial t^2}\,\mathrm{R}(t) = -.1921282720\,10^{-18}\,\frac{\mathrm{Q}(t)\,\left(-1.\,\mathrm{e}^{\left(-.945179584010^{7}\,\mathrm{R}(t)^2\right)}+1.\right)\,kg\_}{\mathrm{R}(t)\,m}$$

$$\frac{\partial}{\partial t}\,\mathrm{Q}(t) = .1929002026\,10^{39}\,\frac{\rho\,R^4\,Z_{atom}\,\mathrm{e}^{\left(-500000.0000\,\frac{\mathrm{x}(t)^2}{\mathrm{m}\_^2}\right)}\,\mathrm{e}^{\left(-.945179584010^{7}\,\frac{\mathrm{y}(t)^2}{\mathrm{m}\_^2}\right)}\,\mathrm{i}(t)}{kg\_\,m\_\,s\_\,A\_\,A_{atom}\,\mathrm{Q}(t)} - .170000000\,10^{8}\,\frac{\mathrm{Q}(t)\,R}{s\_\,m\_} - .9345757880\,10^{25}\,\mathrm{e}^{\left(-56.93916406+.01114000181\,\sqrt{\mathrm{Q}(t)}\right)}$$

Assume initial charge of 100:

```
F := proc(rkf45_x) ... end
```

An odeplot takes ages:

10.3 EOM for Gaussian 2D beam with charge development

$$ForceX2D := (\,x, y\,) \rightarrow -e\_\mathrm{Q}(t)\,\Im(\,\mathrm{Egauss2D\_H}(\,x, y\,)\,)$$

$$ForceY2D := (\,x, y\,) \rightarrow -e\_\mathrm{Q}(t)\,\Re(\,\mathrm{Egauss2D\_H}(\,x, y\,)\,)$$

$$deqnX2D := \frac{\partial^2}{\partial t^2}\,\mathrm{x}(t) = -.2474305362\,10^{-15}\mathrm{Q}(t)\Im\left(\mathrm{w}(\,726.5860976\,\mathrm{x}(t) + 726.5860976\,I\,\mathrm{y}(t)\,)\right)$$

$$- 1.\, e^{\left(-500000.0000\, x(t)^2 - .9451795840\, 10^7\, y(t)^2\right)}\, w(167.1148024\, x(t) + 3159.069990\, I\, y(t))) / m$$

$$deqnY2D := \frac{\partial^2}{\partial t^2} y(t) = -.2474305362\, 10^{-15} Q(t) \Re \left(w(726.5860976\, x(t) + 726.5860976\, I\, y(t)) \right.$$
$$\left. - 1.\, e^{\left(-500000.0000\, x(t)^2 - .9451795840\, 10^7\, y(t)^2\right)}\, w(167.1148024\, x(t) + 3159.069990\, I\, y(t))\right) / m$$

11 FORTRAN output of dustde.f system (x, y , s, vx, vy, vs, q, i, fxkick, fykick)

Equation of motion:

```
 dzdt(4)  = -0.2474305E-15*q*(aimag(CWERF(0.7265861E3*x+0.7265861E3
#*cmplx(0.E0,1.E0)*y))-aimag(exp(-0.5E6*x**2-0.9451796E7*y**2)*CWER
#F(0.1671148E3*x+0.315907E4*cmplx(0.E0,1.E0)*y)))/ mdust
 dzdt(5)  = -0.2474305E-15*q*(Re(CWERF(0.7265861E3*x+0.7265861E3*cm
#plx(0.E0,1.E0)*y))-Re(exp(-0.5E6*x**2-0.9451796E7*y**2)*CWERF(0.16
#71148E3*x+0.315907E4*cmplx(0.E0,1.E0)*y)))/ mdust
 dzdt(7)  = 0.1929002E39* rhodust* Rdust**4* Zatom / Aatom /q*exp(-
#0.5E6*x**2)*exp(-0.9451796E7*y**2)*cur-0.17E8*q* Rdust-0.9345758E2
#5*exp(-0.5693916E2+0.1114E-1*sqrt(q))
```

Beam kicking frequencies:

```
 dzdt(9) = -fxtoy*(f2-f1)*sin(2*0.3141593E1*fsweep*(t-ts))*0.314159
#3E1*fsweep*(1.E0/2.E0+tanh((t-ts)/dts)/2)+fxtoy*(f1/2+f2/2+(f2-f1)
#*cos(2*0.3141593E1*fsweep*(t-ts))/2)*(1-tanh((t-ts)/dts)**2)/dts/2
 dzdt(10) = -(f2-f1)*sin(2*0.3141593E1*fsweep*(t-ts))*0.3141593E1*f
#sweep*(1.E0/2.E0+tanh((t-ts)/dts)/2)+(f1/2+f2/2+(f2-f1)*cos(2*0.31
#41593E1*fsweep*(t-ts))/2)*(1-tanh((t-ts)/dts)**2)/dts/2

 t0 = (f1/2+f2/2+(f2-f1)*cos(2*0.3141593E1*fsweep*(t-ts))/2)*(1.E0/
#2.E0+tanh((t-ts)/dts)/2)
```

Without Heaviside but with intial phase:

```
 dzdt(9) = -fxtoy*(f2-f1)*sin(2*0.3141593E1*fsweep*(t-ts))*0.314159
#3E1*fsweep
 dzdt(10) = -(f2-f1)*sin(2*0.3141593E1*fsweep*(t-ts))*0.3141593E1*f
#sweep
```

Dust constants:

```
 Rdust = 0.1E-5
 mdust = 0.100531E-13
 rhodust = 0.24E4
 Aatom = 60
 Zatom = 30
```

References

Bassetti, M. and Erskine, G. A. (1980), *Closed Expression for the Electrical Field of a Two-dimensional Gaussian Charge*, Technical Report CERN-ISR-TH 80-0, CERN: European Organisation for Nuclear Research

Kheifets, S. (1976), *Potential of a a three dimensional Gauss bunch*, Technical Report PETRA note 119, Deutsches Elektronen Synchrotron

Zimmerman, F. (1994), *Trapped Dust in HERA and Prospects for PEP-II*, Technical Report PEP-II AP Note No.: 8-94, Stanford Linear Accelerator Center